Padasalai.Net – Half Yearly Exam – Model Question Paper

MATHEMATICS

XI STD

TIME : 2 Hrs.30min **Max .Mark 90**

Section A

Answer all the questions **1 x 20 =20**

1

Let R be the universal relation on a set X with more than one element. Then R is

(1) not reflexive (2) not symmetric (3) transitive (4) none of the above

2.

Let $X = \{1, 2, 3, 4\}, Y = \{a, b, c, d\}$ and $f = \{(1, a), (4, b), (2, c), (3, d), (2, d)\}$. Then f is

(1) an one-to-one function (2) an onto function
(3) a function which is not one-to-one (4) not a function

3.

The solution of $5x - 1 < 24$ and $5x + 1 > -24$ is

(1) $(4, 5)$ (2) $(-5, -4)$ (3) $(-5, 5)$ (4) $(-5, 4)$

4,

If $\frac{kx}{(x+2)(x-1)} = \frac{2}{x+2} + \frac{1}{x-1}$, then the value of k is

(1) 1 (2) 2 (3) 3 (4) 4

5.

The triangle of maximum area with constant perimeter $12m$

(1) is an equilateral triangle with side $4m$ (2) is an isosceles triangle with sides $2m, 5m, 5m$
(3) is a triangle with sides $3m, 4m, 5m$ (4) Does not exist.

6.

Which of the following is not true?

(1) $\sin\theta = -\frac{3}{4}$ (2) $\cos\theta = -1$ (3) $\tan\theta = 25$ (4) $\sec\theta = \frac{1}{4}$

7.

The number of ways of choosing 5 cards out of a deck of 52 cards which include at least one king is

(1) ${}^{52}C_5$ (2) ${}^{48}C_5$ (3) ${}^{52}C_5 + {}^{48}C_5$ (4) ${}^{52}C_5 - {}^{48}C_5$.

8.

The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines.

(1) 6 (2) 9 (3) 12 (4) 18

9.

The sum up to n terms of the series $\sqrt{2}+\sqrt{8}+\sqrt{18}+\sqrt{32}+\cdots$ is

(1) $\frac{n(n+1)}{2}$ (2) $2n(n+1)$ (3) $\frac{n(n+1)}{2}$ (4) 1.

10.

If $a, 8, b$ are in AP, $a, 4, b$ are in GP, and if a, x, b are in HP then x is

(1) 2 (2) 1 (3) 4 (4) 16.

11.

If the point $(8,-5)$ lies on the locus $\frac{x^2}{16}-\frac{y^2}{25}=k$, then the value of k is

(1) 0 (2) 1 (3) 2 (4) 3

12.

If one of the lines given by $6x^2-xy+4cy^2=0$ is $3x+4y=0$,, then c equals to

(1) -3 (2) -1 (3) 3 (4) 1

13.

If $\Delta=\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$, then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is

(1) Δ (2) $k\Delta$ (3) $3k\Delta$ (4) $k^3\Delta$

14.

The matrix A satisfying the equation $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}A=\begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ is

(1) $\begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -4 \\ 1 & 0 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 4 \\ 0 & -1 \end{bmatrix}$ (4) $\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}$

15.

If the projection of $5\hat{i}-\hat{j}-3\hat{k}$ on the vector $\hat{i}+3\hat{j}+\lambda\hat{k}$ is same as the projection of $\hat{i}+3\hat{j}+\lambda\hat{k}$ on $5\hat{i}-\hat{j}-3\hat{k}$, then λ is equal to

(1) ± 4 (2) ± 3 (3) ± 5 (4) ± 1

16.

A vector $\overrightarrow{OP}$ makes $60°$ and $45°$ with the positive direction of the x and y axes respectively. Then the angle between $\overrightarrow{OP}$ and the z-axis is

(1) $45°$ (2) $60°$ (3) $90°$ (4) $30°$

17.

If $\lim_{x\to 0}\frac{\sin px}{\tan 3x}=4$, then the value of p is

(1) 6 (2) 9 (3) 12 (4) 4

18.

Let $f:\mathbb{R}\to\mathbb{R}$ be defined by $f(x)=\begin{cases}x & x \text{ is irrational}\\ 1-x & x \text{ is rational}\end{cases}$ then f is

(1) discontinuous at $x=\frac{1}{2}$ (2) continuous at $x=\frac{1}{2}$

(3) continuous everywhere (4) discontinuous everywhere

19.

If $f(x)=\begin{cases}2a-x, & \text{for } -a<x<a\\ 3x-2a & \text{for } x\geq a\end{cases}$, then which one of the following is true?

(1) $f(x)$ is not differentiable at $x=a$ (2) $f(x)$ is discontinuous at $x=a$

(3) $f(x)$ is continuous for all x in $\mathbb{R}$ (4) $f(x)$ is differentiable for all $x\geq a$

20.

The differential coefficient of $\log_{10} x$ with respect to $\log_x 10$ is

(1) 1 (2) $-(\log_{10} x)^2$ (3) $(\log_x 10)^2$ (4) $\frac{x^2}{100}$

Section B

Answer any 7 of the following **2 x 7 =14**

21.

Let f and g be the two functions from R to R defined by f(x) =3x-4 and g(x) =x^2+3 find fog and gof

22.

Discuss the nature of roots of (i) $-x^2+3x+1=0$, (ii) $4x^2-x-2=0$,

23.

Find all the angles between $0°$ and $360°$ which satisfy the equation $\sin^2\theta=\frac{3}{4}$.

24.

A polygon has 90 diagonals. Find the number of its sides?

25. **Find the sum :** $1+\frac{4}{5}+\frac{7}{25}+\frac{10}{125}+\ldots\ldots\ldots\ldots$

26.

If θ is a parameter, find the equation of the locus of a moving point, whose coordinates are $x = a\cos^3\theta, y = a\sin^3\theta$.

27. **Compute** $|A|$ **using Sarrus rule if A =** $\begin{bmatrix} 3 & 4 & 1 \\ 0 & -1 & 2 \\ 5 & -2 & 6 \end{bmatrix}$

28. **For any vector Prove that** $\vec{r} = \left(\vec{r}\bullet\hat{i}\right)+\left(\vec{r}\bullet j\right)+\left(\vec{r}\bullet k\right)$

29.

Prove that $f(x) = 2x^2 + 3x - 5$ is continuous at all points in $\mathbb{R}$

30. **Differentiate with respect to x** $y=\sqrt{x+\sqrt{x}}$

Section C

Answer any 7 questions **3 x 7 = 21**

31.

The total cost of airfare on a given route is comprised of the base cost C and the fuel surcharge S in rupee. Both C and S are functions of the mileage m; $C(m) = 0.4m + 50$ and $S(m) = 0.03m$. Determine a function for the total cost of a ticket in terms of the mileage and find the airfare for flying 1600 miles.

32.

Prove $\log a + \log a^2 + \log a^3 + \cdots + \log a^n = \frac{n(n+1)}{2}\log a.$

33. If the three angles in a triangle are in the ratio 1:2:3 then prove that the corresponding sides are in the ration 1: $\sqrt{3}$ **: 2**

34.

8 women and 6 men are standing in a line.

(i) How many arrangements are possible if any individual can stand in any position?
(ii) In how many arrangements will all 6 men be standing next to one another?
(iii) In how many arrangements will no two men be standing next to one another?

35. Find the value of $\sqrt[3]{65}$ Using Binominal expansion

36

The slope of one of the straight lines $ax^2 + 2hxy + by^2 = 0$ is three times the other, show that $3h^2 = 4ab$.

37.

Find the value of the product ; $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \times \begin{vmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{vmatrix}$.

38.

Find the area of the triangle whose vertices are $A(3, -1, 2)$, $B(1, -1, -3)$ and $C(4, -3, 1)$.

39.

For what value of α is this function $f(x) = \begin{cases} \dfrac{x^4 - 1}{x - 1}, & \text{if } x \neq 1 \\ \alpha, & \text{if } x = 1 \end{cases}$ continuous at $x = 1$?

40. Find the second order derivative of $x^2+y^2=4$

Section C

Answer all the questions **7 x 5 = 35**

41.

Write the values of f at $-3, 5, 2, -1, 0$ if

$$f(x) = \begin{cases} x^2 + x - 5 & \text{if } x \in (-\infty, 0) \\ x^2 + 3x - 2 & \text{if } x \in (3, \infty) \\ x^2 & \text{if } x \in (0, 2) \\ x^2 - 3 & \text{otherwise} \end{cases}$$

(OR)

Determine the region in the plane determined by the inequalities: $2x + 3y \leq 6,\ x + 4y \leq 4,\ x \geq 0, y \geq 0$.

42.

Show that $\sin 12^\circ \sin 48^\circ \sin 54^\circ = \dfrac{1}{8}$.

(OR)

Use induction to prove that $n^3 - 7n + 3$, is divisible by 3, for all natural numbers n.

43.

Two vehicles leave the same place P at the same time moving along two different roads. One vehicle moves at an average speed of $60km/hr$ and the other vehicle moves at an average speed of $80\ km/hr$. After half an hour the vehicle reach the destinations A and B. If AB subtends $60°$ at the initial point P, then find AB.

(OR)

In a certain town, a viral disease caused severe health hazards upon its people disturbing their normal life. It was found that on each day, the virus which caused the disease spread in Geometric Progression. The amount of infectious virus particle gets doubled each day, being 5 particles on the first day. Find the day when the infectious virus particles just grow over 1,50,000 units?

44.

Show that the equation $4x^2 + 4xy + y^2 - 6x - 3y - 4 = 0$ represents a pair of parallel lines. Find the distance between them.

(OR)

If the binomial coefficients of three consecutive terms in the expansion of $(a+x)^n$ are in the ratio $1 : 7 : 42$, then find n.

45. **Prove that** $\begin{vmatrix} 1 & x^2 & x^3 \\ 1 & y^2 & y^3 \\ 1 & z & z^3 \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$ (Using factor theorem)

(OR)

If $\vec{a},\vec{b},\vec{c}$ are position vectors of the vertices A, B, C of a triangle ABC, show that the area of the triangle ABC is $\frac{1}{2}|\vec{a}\times\vec{b}+\vec{b}\times\vec{c}+\vec{c}\times\vec{a}|$. Also deduce the condition for collinearity of the points A, B, and C.

46.

Prove that the line segments joining the midpoints of the adjacent sides of a quadrilateral form a parallelogram.

(OR)

Prove that $\lim_{\theta\to 0}\frac{\sin\theta}{\theta}=1$ **Using the result Find the value of** $\lim_{\theta\to 0}\frac{1-\cos\theta}{\theta}$

47.

If $y = (\cos^{-1} x)^2$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. Hence find y_2 when $x = 0$

(OR)

A committee of 7 peoples has to be formed from 8 men and 4 women. In how many ways can this be done when the committee consists of

(i) exactly 3 women?
(ii) at least 3 women?
(iii) at most 3 women?

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