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Reg. No.

110141

Second Mid-Term Test - 2018

Time : 1.30 hrs.

MATHEMATICS

Max. Marks : 50

PART - A

I. Answer all the following

10 x 1 = 10

1. If $2x + \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 8 \\ 7 & 2 \end{pmatrix}$ find the matrix X.

a) $\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ b) $\begin{pmatrix} 1 & -3 \\ 2 & -1 \end{pmatrix}$ c) $\begin{pmatrix} 2 & -6 \\ 4 & -2 \end{pmatrix}$ d) $\begin{pmatrix} 2 & 6 \\ 4 & -2 \end{pmatrix}$

2. If $A = \begin{pmatrix} 1 & -1 \\ 2 & -1 \end{pmatrix}$ $B = \begin{pmatrix} a & 1 \\ b & -1 \end{pmatrix}$ and $(A + B)^2 = A^2 + B^2$ then the value of a and b are

a) $a = 1, b = 4$ b) $a = 4, b = 1$ c) $a = 0, b = 4$ d) $a = 2, b = 4$

3. For any square matrix A with real numbers the matrix $A - A^T$ is
 a) diagonal matrix b) symmetric matrix c) skew symmetric matrix d) null matrix

4. If $\Delta = \begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix}$ then $\begin{vmatrix} ka & kb & kc \\ kx & ky & kz \\ kp & kq & kr \end{vmatrix}$ is

a) $K\Delta$ b) $K^2\Delta$ c) $K^3\Delta$ d) $3K\Delta$

5. The value of $\begin{vmatrix} \log_3 64 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix}$ is a) 3 b) 9 c) $\frac{9}{2}$ d) $\frac{15}{2}$

6. The value of $\vec{AB} + \vec{BC} + \vec{DA} + \vec{CD}$ is

a) \vec{AD} b) \vec{O} c) \vec{CA} d) $-\vec{AD}$

7. If the position vector of A is $3\vec{i} + 2\vec{j} - \vec{k}$ and

$\vec{AB} = -\vec{i} + \vec{j} + 3\vec{k}$ then position vector of B is

- a) $2\vec{i} + 2\vec{j} + 2\vec{k}$ b) $2\vec{i} + 3\vec{j} + 2\vec{k}$ c) $4\vec{i} + \vec{j} - 4\vec{k}$
d) $\vec{i} - \vec{j} - 3\vec{k}$

8. If $\lambda\vec{i} + 2\lambda\vec{j} + 2\lambda\vec{k}$ is a unit vector then the value of λ is

- a) $\frac{1}{3}$ b) $\frac{1}{4}$ c) $\frac{1}{9}$ d) $\frac{1}{2}$

9. If \vec{a} and \vec{b} having some magnitude and angle between

them is 60° and their scalar product is $\frac{1}{2}$ then $|\vec{a}|$ is

- a) 1 b) 2 c) 3 d) 7

10. If \vec{a} and \vec{b} inclined at an angle 120° and $|\vec{a}| = 1$,
 $|\vec{b}| = 2$ then $(\vec{a} + 2\vec{b}) \times (2\vec{a} - \vec{b})$ a) $-\vec{i}$ b) $15\vec{i}$ c) $-3\vec{i}$ d) 0

PART - B

Answer any five, Q.No.17 is compulsory.

5 x 2 = 10

11. If $A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ find A^4 .

12. Construct the matrix $A = [a_{ij}]_{3 \times 3}$ where $a_{ij} = i - j$. State whether A is a symmetric or skew symmetric matrix.

13. Without expanding prove that

$$\begin{vmatrix} s & a^2 & b^2 + c^2 \\ s & b^2 & c^2 + a^2 \\ s & c^2 & a^2 + b^2 \end{vmatrix} = 0$$

14. Show that

$$\begin{vmatrix} 0 & c & b \\ c & 0 & a \\ b & a & 0 \end{vmatrix}^2 = \begin{vmatrix} b^2 + c^2 & ab & ab \\ ab & c^2 + a^2 & bc \\ ac & bc & a^2 + b^2 \end{vmatrix}$$

15. If A and B be two points with position vectors $2\vec{a} + 4\vec{b}$ and $2\vec{a} - 8\vec{b}$. Find the position vectors of the points which divide the line segment joining A and B in the ratio 1 : 3 internally and externally.

16. Show that $\vec{a} = 2\vec{i} + 3\vec{j} + 6\vec{k}$, $\vec{b} = 6\vec{i} + 2\vec{j} - 3\vec{k}$ and $\vec{c} = 3\vec{i} - 6\vec{j} + 2\vec{k}$ are mutually orthogonal.

17. For any two vectors \vec{a} and \vec{b} prove that

$$|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$$

PART - C

Answer any five, Q.No.24 is compulsory.

5 x 3 = 15

18. If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ x & 2 & y \end{pmatrix}$ is such that $A \cdot A^T = 5I$, then find the value of x and y.

19. Prove that $\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc \left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right)$

20. Prove that $\begin{vmatrix} 1 & x & x^2 \\ x & 1 & x \\ x & x & 1 \end{vmatrix} = \begin{vmatrix} 1-2x^2 & -x^2 & -x^2 \\ -x^2 & -1 & x^2-2x \\ -x^2 & x^2-2x & -1 \end{vmatrix}$

21. If D and E are the midpoints of the sides AB and AC of ΔABC

the prove that $\vec{BE} + \vec{DC} = \frac{3}{2} \vec{BC}$

22. The position vector of the vertices of a triangle are

$\vec{i} + 2\vec{j} + 3\vec{k}$, $3\vec{i} - 4\vec{j} + 5\vec{k}$ and $-2\vec{i} + 3\vec{j} - 7\vec{k}$

Find the perimeter of the triangle.

23. If $|\vec{a}| = 3$, $|\vec{b}| = 4$; $|\vec{c}| = 5$ and $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ find

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$$

24. Find the area of the triangle whose vertices are A(3, -1, 2)
B(1, -1, -3) and C(4, -3, 1)

PART - D

Answer all the following

3 x 5 = 15

25. a) Prove that
$$\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix} = (x - y)(y - z)(z - x)$$

(OR)

b) Using factor theorem prove
$$\begin{vmatrix} b+c & a-c & a-b \\ b-c & c+a & b-a \\ c-b & c-a & a+b \end{vmatrix} = 8abc$$

26. a) Using vector methods prove that the medians of a triangle are concurrent.

(OR)

b) If ABCD is a quadrilateral and E and F are the midpoints of AC and BD respectively, then prove that

$$\vec{AB} + \vec{AD} + \vec{CB} + \vec{CD} = 4\vec{EF}$$

27. a) Show that the vectors $5\vec{i} + 6\vec{j} + 7\vec{k}$, $7\vec{i} - 8\vec{j} + 9\vec{k}$ and $3\vec{i} + 20\vec{j} + 5\vec{k}$ are coplanar. (OR)

b) If \vec{a} , \vec{b} , \vec{c} be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and the angle between \vec{b} and \vec{c} is $\frac{\pi}{6}$ then prove that

$$\vec{a} = \pm 2(\vec{b} \times \vec{c})$$